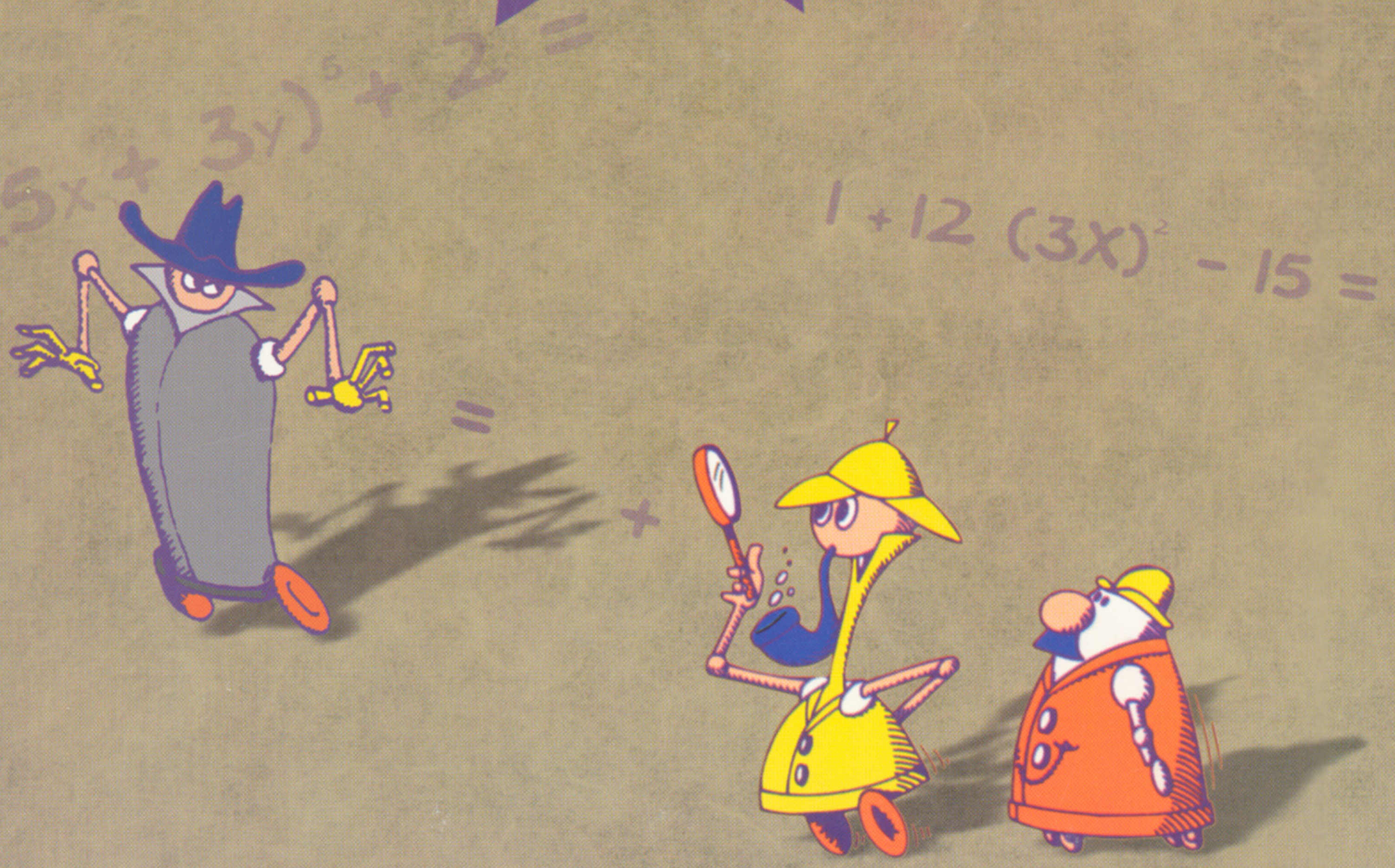
THE POWER OF ALGEBRA



TEACHER'S GUIDE

CORRELATED TO THE

NCTM PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS

THE POWER OF ALGEBRA

PROGRAM ONE

<u>INVERSE OPERATIONS</u>

A. SYNOPSIS

This program will:

Explore the nature of variables: how letters, etc., are used as placeholders for numbers.

Introduce the process of solving algebraic equations through forming equivalent equations.

Demonstrate how algebra is used in everyday life to solve problems.

NCTM STANDARDS ADDRESSED IN PROGRAM ONE

STANDARD	GRADES 6-8, 9-12
Number and Operation	Students should understand meanings of operations and how they relate to one another
Patterns, Functions, & Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols.
Geometry	Students should use visualization, spatial reasoning, and geometric modeling to solve problems.

B. VOCABULARY

NOTE TO THE TEACHER: Every formula is an equation.

VARIABLE..... a symbol, usually a letter, used to represent any element of a given set of more than one element, a placeholder for a number

OPERATION. . . addition, subtraction, multiplication and/or division of real numbers.

EQUATION........ any statement involving the relationship "is equal to ..."

SOLUTION .... any number which makes the equation true.

EQUIVALENT EQUATIONS ..... equations which have the same solutions.

INVERSE OPERATIONS . addition and subtraction are **inverse operations** of each other -- they "undo" each other; multiplication and division are **inverse operations** of each other -- they "undo" each other.

C. PRE-PROGRAM ACTIVITY

1. Have two students of comparable height stand back to back in front of the class. Ask "Are their heights equal? How would we write this as an equation?" (Example: Height of J = Height of K)

   Then have one student stand on a book. Ask: "Are their heights still equal? What do we do to make their heights the same?"

D. DISCUSSION QUESTIONS

1. What is the main idea of the program?

2. Can you think of any real-life applications or examples of inverse operations?

3. Summarize what you learned in the lesson.

4. Give examples of variables in everyday life.
   (Examples: electric meter, a person's weight, odometer reading, checkbook balance, speedometer reading, change from a dollar after a purchase, etc.)

E. FOLLOW-UP ACTIVITIES

1. Balancing the Scale

For this activity, you will need: a pan balance or a beam balance and appropriate weights (or common classroom objects). Show how the weight of two or more objects can be equal to the weight of a single object. Write this as an equation. (Example: 2 + 3 = 5)

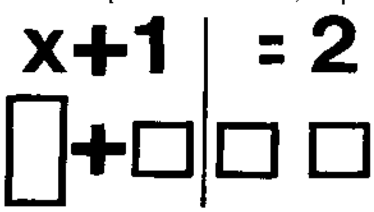
Place additional weight(s) on one side so the scale becomes unbalanced. Discuss what must be done to balance the scales. Write this as an equation. (Example: 2 + 3 + 1 = 5 + 1)

2. Using Algebra Tiles

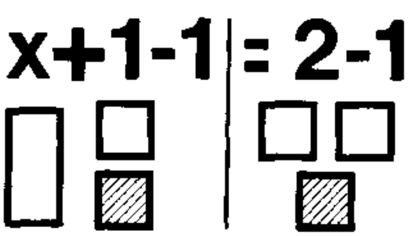
a. An Example to Work Together

Remember that

For the equation **x + 1 = 2**, display the blocks this way:



To solve the equation, you must get your X block alone. To do this, add a **negative 1** block (   ) to both sides. This gives you



While working in small groups, have the students use algebra tiles to model the following equations, and then use the model to solve the equation. After all groups have completed the modeling activity, then the students should model the process of solving each equation on the overhead projector. Be sure to allow students to show alternative methods of solving, if they modeled differently than what was presented on the overhead.

3x – 1 = 8	**x + 4 = 2x – 3**
2x + 1 = 2	**5 = 4x -3**

3. Using the "Golden Rule"

NOTE TO THE TEACHER: Division by zero is impossible, since $a / 0 = b$ means $0 \cdot b = a$ but $0 \cdot b \neq a$. (EXAMPLE: $10 / 2 = 5$ because $5 \cdot 2 = 10$, but $5 / 0 \neq 5$ because $0 \cdot 5 \neq 5$. The answer is undefined.)

The **"GOLDEN RULE"** of equivalent equations is:

DO UNTO ONE SIDE OF AN EQUATION AS YOU WOULD DO TO THE OTHER.

This means when you add, subtract, multiply or divide (except when dividing by zero) on one side of an equation, you must perform the same operation on the other side.

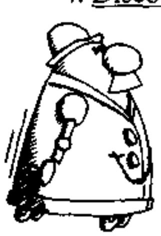
a. For this activity, have the students work with a partner to discuss what they would do to solve each of the following.

$x + 2 = 5$	(Subtract 2 from both sides)
$a - 7.2 = 12.5$	(Add 7.2 to both sides)
$3x = 10$	(Divide both sides by 3)
$y / 4 = 5$	(Multiply both sides by 4)

b. Students should work with their partner to examine the equivalent equations and determine what was done to the first equation to produce the second.

$2x - 7 = 21$	$31 - x = 120$
$2x = 28$	$-x = 89$

4. Discovering Equivalent Equations



NOTE TO THE TEACHER: It is important to understand that in the process of applying this method of solving equations, you move from one equation to an equivalent equation at each step.

a. Have students work in groups to determine which of the following equations are equivalent and why.

$9 = x$	$x - 1 = 8$
$x / 3 = 3$	$x + 2 = 7$
$5x = 45$	$3x + 2 = 29$
$2x = 16$	

b. Have students work in groups to determine why the following equations are not equivalent.

a = 9

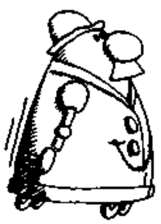
a - 1 = 10

2a + 5 = 40

a / 5 = 1

7a = 6

5. Solving the Equations Which Appeared in the Program



NOTE TO THE TEACHER: When variables are on both sides of the equation, the "Golden Rule" works on the variable, too.

a. Have students work in groups to determine how the following equations would be solved. They should explain how the "Golden Rule" applies and how they used inverse operations to solve each.

x + 150 = 180

x – 150 = 180

150 • x = 180

x ÷ 150 = 180

4x = x + 12

10x = 12(x - 1/4)

10x = 12x - 3

There is usually more than one way to correctly solve an equation, although some ways are more efficient than others. Encourage students to show the different methods they used to solve the equations.

b. Since a solution is any number which makes the equation true, students can check their answers and be sure that they are correct by substituting the value for the variable into the original equation. You might want to allow students to use a calculator to make checking a quick and efficient process. Have students check their answers to part a.

c. Below are four charts used in the program to explain the **Rate • Time = Distance** problem Holmes solved. You may want to review the problem and set up the equation for the students to solve with a partner.

	RATE ·	TIME =	DISTANCE
		X	
		$X-\frac{1}{4}$	

	RATE ·	TIME =	DISTANCE
	10	X	
	12	$X-\frac{1}{4}$	

	RATE ·	TIME =	DISTANCE
	10	X	$10X$
	12	$X-\frac{1}{4}$	

	RATE ·	TIME =	DISTANCE
	10	X	$10X$
	12	$X-\frac{1}{4}$	$12(X-\frac{1}{4})$

F. STUDENT WORKSHEET

INVERSE OPERATIONS **Name**

Use inverse operations to solve the following. Be sure to check your answers.

1. $32.5 + y = 60$
2. $x / 22 = 1/2$
3. $a - 543 = 76$
4. $17x = 136$
5. $3y + 2 = 27$
6. $2a - 3 = a + 2$

Write an equation for the following situation and solve.

A plumber charges $45 **for a service call plus** $22 **per hour. If the bill from the plumber was $133, how many hours did he work?**

Program One Inverse Operations

Student Worksheet F

1. $y = 27.5$
2. $x = 11$
3. $a = 619$
4. $x = 8$
5. $y = 8\frac{1}{3}$
6. $a = 5$
7. $133 = 45 + 22h$

   $h = 4$

THE POWER OF ALGEBRA

PROGRAM TWO

A. SYNOPSIS

This program will:

Define the order of operations for multiplication, division, addition and subtraction.

Define the order of operations within grouping symbols relative to multiplication, division, addition and subtraction.

Define exponents and the order of operations with exponents and grouping symbols.

Discuss how these rules may be used to simplify algebraic problems.

NCTM STANDARDS ADDRESSED IN PROGRAM TWO

STANDARD	GRADES 6 - 8
Number and Operations	Students should extend understanding of operations to include nonnegative whole number exponents.
Number and Operations	Students should compute fluently and make reasonable estimates.

B. VOCABULARY

FACTOR...... If two numbers a and b are multiplied together to form a product ab, then a and b are called the **factors** of the product ab.

POWER........ If the number a is used as a factor n times, the product may be written as a". A product of the form a" is called a **power.**

In the **power** a", **a** is called the **base** of the power and n is called the **exponent** of the power.

NOTE TO THE TEACHER:

a) In the power a", n is called a superscript.

b) b) x_n is read "x sub n" where sub stands for subscript and is not the same as the power x".

OPERATION........ Addition, subtraction, multiplication or division of real numbers.

C. FOLLOW-UP ACTIVITY

1. "Please Excuse My Dear Aunt Sally." This phrase will help students remember the rules for order of operations. Put on the chalk board:

P

E

M D

A S

Say "Please Excuse My Dear Aunt Sally. This stands for

Parentheses

Exponents

Multiplication and Division

Addition and Subtraction."

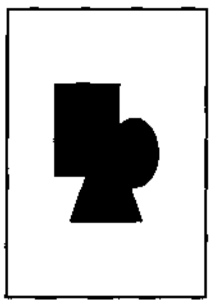
Emphasize that multiplication and division have the same priority; that is, they are done from left to right as they occur in the expression. Addition and subtraction also have the same priority. That is why the letters which stand for multiplication / division and addition / subtraction are written on the same line.

2. Working with a partner, the students should use the following pair of problems to determine how the addition of parentheses changes the problem.

$2 + 3 \bullet 5 - 8 \div 2^2$ $\quad\quad\quad\quad$ $(2 + 3)5 - (8^2 \div 2)^2$

3. Explain the difference between -2^4 and $(-2)^4$.

-2^4 means	$(-2)^4$ means
$(-1)(2)(2)(2)(2)$	$(-2)(-2)(-2)(-2)$
-16	16



NOTE TO THE TEACHER: You may need to explain all the different ways that the operation of multiplication can be shown in algebra.

Ex. $\quad$ **2 x 3 (rarely used due to confusion with variable)**

$2 \bullet 3$

$2(3)$

$(2)(3)$

4. Review the rules of the Order of Operations.
   a) First, perform the exponential operations.

   Ex. $\quad 50 - 2^2 \bullet (2 + 3) \div 10$

   $50 - 4 \bullet (2 + 3) \div 10$

   b) Second, perform the operations inside grouping symbols.

   Ex. $\quad 50 - 4 \bullet (2 + 3) \div 10$

   $50 - 4 \bullet 5 \div 10$

   c) Third, perform multiplication and division as they occur, starting on the left of the expression and moving to the right.

   Ex. $\quad 50 - 4 \bullet 5 \div 10$

   $50 - 20 \div 10$

   $50 - 2$

d) Fourth, perform addition and subtraction as they occur, starting on the left of the expression and moving to the right.

Ex. $50-2 = 48$

5. Have the students show that a different result would have been obtained in step 4 if a different order of operations had been applied.

6. Solve the equation that appeared in the program.

$$(8 \div 16^2) = (D^2 \div 20^2)$$

7. In the video, why is $x = 3 + 4 \bullet 2$ equal to **11** and not equal to **14**? Also, why is $x = 3 + (4 \div 2)$ equal to **5** and not equal to **3.5**?

D. DISCUSSION QUESTIONS

1. Show how exponents can be used to write a product in simpler form.

   Ex. $2 \bullet 2 \bullet 2 \bullet 5 \bullet 5 \bullet 7 \bullet 7 \bullet 7 \bullet 7 = 2^3 \bullet 5^2 \bullet 7^4$

2. Why is $8 - 6x + 5$ not equal to $2x + 5$?

3. The solution of the equation $2 + (4^2 + x) + 6x \div 3 = 26$ is a) **7**, b) **2.68**, c) **8/3**, d) **106.5** Why?

NOTE TO THE TEACHER:

1. **a^n is read as "a to the n^{th} power." For example a^2 is read "a squared," a^3 is read "a cubed," and a^6 is read "a to the sixth power."**

2. **In discussing factors, it should be noted that the product ab is a multiple of a and also a multiple of b. We also call a and b divisors of the number ab.**

E. <u>STUDENT WORKSHEET</u>

ORDER OF OPERATIONS #1

Compute each of the following:

1. $2 + 3 \bullet 6$
2. $(2 + 3)\,6$
3. $10 \div 2 \bullet 3$
4. $10 \div (2 \bullet 3)$
5. $16 - 3 + 8$
6. $16 - (3 + 8)$
7. $6 + 24 + 6 \bullet 2 - 4$
8. $[\,(6 + 24) \div 6\,] \bullet 2 - 4$
9. $12 - 3 + 18 \div 3^2$
10. $12 - 3 + (18 \div 3)^2$

Pick a problem from the first column and compare it to its partner in the second column. Did the answers change even though the problems were using the same numbers and operations? Explain.

E. STUDENT WORKSHEET

ORDER OF OPERATIONS #2 **Name**

Simplify the following expressions:

1. $1.6 + (6)^2 \div 9$
2. $½ + (2^2 - 1)$
3. $5.3 + (3.2)^2 \bullet 0.1$
4. $81 \div (3)^2 + (-4)^2$

Get out your calculator and determine if the calculator follows Order of Operations. Simplify the expression below by hand and record your answer. Type the entire expression into your calculator before pressing the "=" key. Does your calculator follow Order of Operations? Explain.

$$5^2 - (13 + 8) \div 3$$

Now use your calculator to simplify the following expression.

$$356.2 + (1.25)^2 + 5$$

Program Two The Order of Operations

Student Worksheet E #1

1. 20

2. 30

3. 15

4. $\frac{5}{3}$ or $1\frac{2}{3}$

5. 21

6. 5

7. 10

8. 6

9. 11

10. 45

Student Worksheet E #2

1. 5.6
2. $\frac{1}{6}$
3. 6.324
4. 25
5. 18 , Most scientific and graphing calculators follow the Order of Operations.
6. 356.5125

THE POWER OF ALGEBRA

PROGRAM THREE

BASIC PROPERTIES

A. SYNOPSIS

This program will:

Explore the properties of addition and multiplication of numbers and variables.

Explore a property of multiplication of variables and numbers that
is linked with the addition of variables and numbers.

Discuss how these properties may be used to simplify algebraic problems.

NCTM STANDARDS ADDRESSED IN PROGRAM THREE

STANDARD	GRADES 6 – 8, 9 – 12
Number and Operations	Students should understand meanings of operations and how they relate to one another.
Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols.
Geometry	Students should use visualization, spatial reasoning, and geometric modeling to solve problems.

B. VOCABULARY

COMMUTATIVE PROPERTY OF ADDITION .... $\mathbf{a + b = b + a}$

COMMUTATIVE PROPERTY OF MULTIPLICATION .... $\mathbf{a \bullet b = b \bullet a}$

ASSOCIATIVE PROPERTY OF ADDITION .... $\mathbf{a + (b + c) = (a + b) + c}$

ASSOCIATIVE PROPERTY OF MULTIPLICATION .... $\mathbf{a\ (b \bullet c) = (a \bullet b) \bullet c}$

DISTRIBUTIVE PROPERTY OF MULTIPLICATION. .... $\mathbf{a \bullet (b + c) = a \bullet b + a \bullet c}$

LIKE TERMS..... **like terms** of an algebraic expression are the terms with exactly the same variable part.
Ex. **3a and 6a are like terms, but 3a and** $\mathbf{6a^2}$ are <u>not</u> **like terms.**

SIMPLIFIED ...... an algebraic expression is **simplified** when like terms are combined by adding their numerical coefficients.

C. FOLLOW-UP ACTIVITIES

1. Show that the **commutative property of addition** verifies that $\mathbf{a = a + 0 = 0 + a}$ for all numbers **a**. Because of this, **0** is defined as the **identity for addition.** Show that the **commutative property of addition** verifies that $\mathbf{0 = a + (-a) = (-a) + a}$. Thus **-a** is called the **additive inverse** of **a.**

2. Students should work with a partner to show how the **commutative and associative properties of addition** may be used to simplify the following algebraic expressions.

   $\mathbf{3a - 5 + 6a + 10}$

   $\mathbf{(4y + 6) + (y + 7)}$

   $\mathbf{2x^2 - y + x^2 + 7}$

3. Show that the **commutative property of multiplication** verifies that

$$\mathbf{a = 1 \bullet a = a \bullet 1}$$

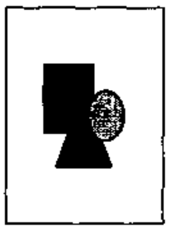
Thus, **1** is called the **identity for multiplication**.

Also, show that the **commutative property of multiplication** verifies that

$$\mathbf{1 = a \bullet (1 / a) = (1 / a) \bullet a}$$

if **a** is not equal to **0**.

Therefore, **(1 / a)** is called the **reciprocal of a** or the **multiplicative inverse** of **a**.



NOTE TO THE TEACHER: It is important that a is not equal to zero since (1 / 0) is not defined.

4. Students should work with a partner to show how the **commutative and associative properties of multiplication** may be used to simplify the following algebraic expressions.

   $\mathbf{(6x) \bullet (3)}$
   $\mathbf{5 \bullet x \bullet 6}$

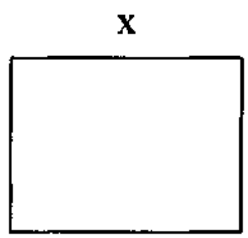
   Students should work with a partner to show how the **distributive property of multiplication with respect to addition** may be used to remove grouping symbols in the following algebraic expressions.

   $\mathbf{6(x + 7)}$

   $\mathbf{2(x^2 - 5x + 10)}$

   $\mathbf{5(y + 7) + 6(2y + 4)}$

6. Use an area model to illustrate the **commutative property of multiplication.**



Is the area of this rectangle XY or YX?

D. <u>DISCUSSION QUESTIONS</u>

1. Is the **commutative property** true for subtraction? If not, give a counter example.

2. Is the **commutative property** true for multiplication? If not, give a counter example.

3. Is $\mathbf{(a - b) - c = a - (b - c)}$? If not, give a counter example.

4. Is $\mathbf{(a + b) \div c = a \div (b \div c)}$? If not, give a counter example.

5. What conclusions can you draw from (3) and (4)?

6. Is $\mathbf{-(2x + 5) = -2x - 5}$? Why?

7. In the earlier example, why are $\mathbf{3a}$ and $\mathbf{6a^2}$ not like terms?

NOTE TO THE TEACHER:

a. After discussion question (6), it should be stressed that a minus sign before a grouping symbol means that every item inside the grouping symbol must be multiplied by –1.

b. After discussing the additive and multiplicative identities and the additive and multiplicative inverses, it should be stressed that each of these is unique. That is, there is only one of each.

c. A <u>counter-example</u> is one example to show that a given statement is not true.

Ex. Statement: All odd numbers are divisible by 3. (False.)

<u>Counter-example:</u> 11 is not divisible by 3.

8. How do the commutative, associative, and distributive properties relate to the Order of Operations?

STUDENT WORKSHEET

BASIC PROPERTIES #1 Name_______________________________

Simplify the following:

1. $2(a^2 - 5) + (3 - a) \bullet 6a$

2. $(x^2 + y^2) + (-x^2 - y^2)$

3. $3y(2x - 5) + x(2y - 5)$

4. $(x + y) \cdot (x - y)$

5. $(t - 5) \cdot (t - 6)$

6. $2x - 7\,[4x - 2(x + 10)]$

E. STUDENT WORKSHEET (continued)

BASIC PROPERTIES #2

State the rule(s) which justifies the following statements:

1. $7 + (2 + 5) = (7 + 5) + 2$

2. $6 + 10 = 10 + 6$

3. $(x + y) \bullet 2 = 2x + 2y$

4. $(x + y) + 7 = (x + 7) + y$

5. $t \bullet (xy) = x \bullet (yt)$

Write a numerical example to illustrate each of the following properties:

1. **Commutative Property of Multiplication**

2. **Associative Property of Addition**

3. **Distributive Property of Multiplication over Addition**

Explain why the Commutative Property of Addition is useful in simplifying this problem.

$x + 5 + 3x$

Show how you could use the Commutative Property of Addition to simplify this problem mentally.

$25 + 32.68 + 75$

Program Three Basic Operations

Student Worksheet E #1

1. $-4a^2 + 18a - 10$

2. 0

3. $-5x + 8xy - 15y$

4. $x^2 - y^2$

5. $t^2 - 11t + 30$

6. $-12x + 140$

Student Worksheet E #2

1. Associative Property for Addition & Commutative Property for Addition

2. Commutative Property for Addition

3. Distributive Property

4. Associative Property for Addition & Commutative Property for Addition

5. Associative & Commutative Properties for Multiplication

Section 2

1. $3 \bullet 5 = 5 \bullet 3$

2. $(x + 5) + 72 = x + (5 + 72)$

3. $-2(x + 6) = -2x + -12$

Section 3

The Commutative Property of Addition is useful in simplifying the problem because it allows you to change the order and add the x and the 3x .

Section 4

The Commutative Property of Addition allows you to add 25 and 75 first. Then add 100 to 32.68 to get 132.68.

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PROGRAM FOUR

THE POSITIVE AND NEGATIVE NUMBERS

A. SYNOPSIS

This program will:

Explain the real number line and the effect of directions on the set of real numbers.

Explore the relationship between positive numbers, negative numbers, and zero.

Demonstrate the need and uses of positive and negative numbers.

Use the real number line to define subtraction of real numbers.

Define subtraction of signed numbers.

NCTM STANDARDS ADDRESSED IN PROGRAM FOUR

STANDARD	GRADES 6 – 8, 9 – 12
Number and Operations	Students should understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Number and Operations	Students should understand and use the inverse relationships of addition and subtraction and multiplication and division.
Number and Operations	Students should compute fluently and make reasonable estimates.

B. VOCABULARY

ZERO OR ORIGIN ON A NUMBER LINE............The point on the number line that is chosen as the starting point.

POSITIVE NUMBERS...........Those numbers which are coordinates of points to the right (or above) zero on the number line.

NEGATIVE NUMBERS.........Those numbers which are coordinates of points to the left (or below) zero on the number line.

SIGNED NUMBERS..............The set consisting of all positive numbers, all negative numbers, and zero.

OPPOSITE OF A NUMBER..On a number line, any number can be paired with another number that is the same distance from **0** and on the opposite side of **0**. Such a pair of numbers is called **opposite.** Example: 3, -3

SUBTRACTION OF SIGNED NUMBERS..............To subtract one **signed number** from another, add the opposite of the number to be subtracted to the other number; i.e. if **a** and **b** are any two **signed numbers,** then **a - b = a + (-b)**.

GREATER THAN...................Lies to the right or above.

LESS THAN.............................Lies to the left or below.

NOTE TO THE TEACHER:

a) Uses of the Symbol (-).

If a and b represent any two real numbers, then in the expression a - (-b) , the symbol (-) is used in two different ways. The first (-) which is located between the two real numbers indicates the operation of subtraction. The second (-) which is part of the number (-b) indicates the opposite of b and -(-b) means the opposite of (-b) which is b.

b) Notice that it is always possible to subtract one signed number from another. This can be summarized by saying that the set of signed numbers closed with respect to subtraction.

C. FOLLOW-UP ACTIVITIES

1. Use a football field to show the position of a team who had the ball on the 30-yard line, gained 8 yards on the first play, lost 3 yards on the second play, and gained 5 yards on the third play.

2. Show addition, subtraction, multiplication and division on the number line.

3. Show the rules for addition, subtraction, multiplication and division of signed numbers.

4. Show that for any real number **a, a + (-a) = 0** and **(-a) + a = 0.** Thus each number in the pair **(a, -a)** is called the **opposite** or the **additive** inverse or the **negative** of the other number and **0** is called the **additive identity** or the **identity element of addition.**

5. Show the multiplication property of **0**; i.e. for every signed number **a, a • 0 = 0** and **0 • a = 0**. This concept is very important in solving quadratic equations.

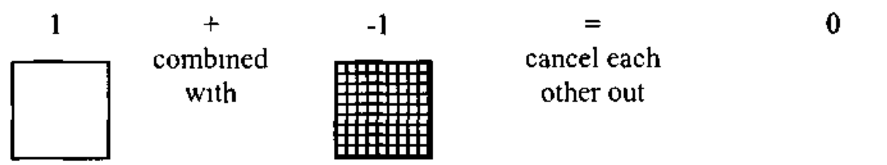
6. Show the multiplication property of **1**, i.e. for every signed number **a, a • 1 = a** and **1 • a = a.** Explain that **1** is called the **identity element of multiplication** or the **multiplicative identity.** This is very important in division of signed numbers. Also show that for every signed number **a, a(-1) = -a** and **(-1)a = -a.**

7. Show division of signed numbers as a fraction or as multiplying by the reciprocal, i.e. for any two signed numbers **a** and **b**, **b ≠ 0, a ÷ b** means **(a/b)** or **a(1/b)**. Actually, **a ÷ b** means find a number which when multiplied by **b** gives **a**, i.e. **a ÷ b** or **(a/b)** means find **c** such that **cb = a.**

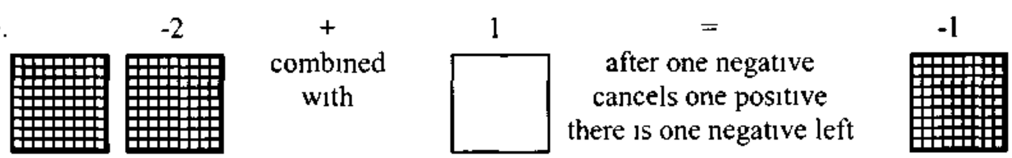
8. Use (5) and (7) to show why division by **0** is impossible, since **a ÷ 0** means find the number which when multiplied by **0** gives **a.** There is no such number since the product of any signed number and **0** is **0.**

9. Use two-color chips or the small squares from your algebra blocks to model addition and subtraction integers.

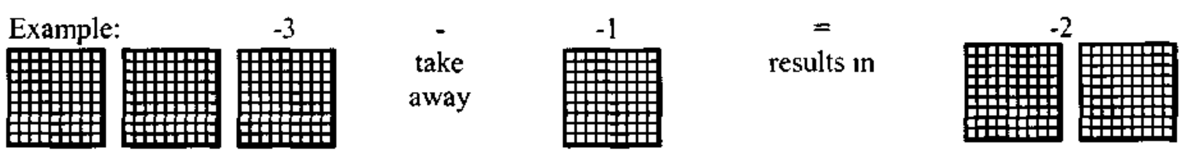
Example:



Example.



Example:



This example could also be done by adding in a +1 instead of subtracting a – 1.

D. STUDENT WORKSHEET

THE POSITIVE AND NEGATIVE NUMBERS #1 **Name**

1. Complete the chart with a signed number to indicate the difference in actual weight and normal weight for each child.

	Christy	Garrett	Ashley	B.J.	Erin	Liz	Jason	Annie
Normal Weight	21	16	39	42	19	40	101	105
Actual Weight	24	21	39	36	23	46	88	97
Difference	+3							

2. Write the symbol to represent the opposite of each number.

a. **+ 4 ½** ______

b. **-8.4** ______

c. **- (-5)** ______

d. **(8 - 8)** ______

e. **- ¾** ______

f. **-[-(5 + 2)]** ______

3. Select the greater of the two numbers.

a. **-4** **-9**

b. **-8** **1**

c. **0** **-10**

d. **-1** **1**

4. Write an inequality to show the relationship between the elevations of 300 feet below sea level and of 48 feet below sea level.

D. STUDENT WORKSHEET (continued)

THE POSITIVE AND NEGATIVE NUMBERS #1

Name ____________________

5. Simplify each expression:

   a. **[(-2 + 3) + (-1)]**

   b. **– [- (-5)]**

   c. **–3.4 + -2.8**

   d. **(-7) - (-23)**

   e. **(–18) ÷ (-2)**

   f. **½ (-16x)**

6. What real situation does each of the following represent:

   -2 in golf

   -15 ½ in the stock market

   -3 in an elevator

   Write an expression and then simplify to answer each question.

   a. **Joseph saved $20 in March, $5 in April, and spent $7 in May. How much money did he have at the end of May?**

   b. **From a full basket of apples, the farmer threw out 14 rotten ones. If there were 48 apples remaining in the basket, how many were in the full basket?**

   c. **A half hour TV show usually has 4 ½ minutes of commercials? How much time does that leave for the show?**

   d. **A football team gained 7 yards on the first play, lost 4 yards on the second play, and gained 8 yards on the third play. Did the team make a first down in those three plays? Explain.**

Program Four The Positive and Negative Numbers

Student Worksheet D #1

1. Differences Garret +5 , Ashley 0 , B.J. –6 , Eric +4 , Liz +6 , Jason -13 , Annie –8

2. a) - b) + c) - d) 0 e) + f) -

3. a) -4 b) 1 c) 0 d) 1

4. $-300 < -48$

5. a) 0 b) -5 c) -6.2 d) 16 e) 9 f) -8x

6. 2 under par, a drop of 15 ½ points, go down 3 floors or the 3rd floor below street level

7. a) $20 + 5 - 7 = \$18$ b) $48 + 14 = 62$ apples c) $30 - 4\frac{1}{2} = 25\frac{1}{2}$ minutes

   d) $7 + -4 + 8 = 11$ Yes, a first down is achieved after a gain of 10 yards.

THE POWER OF ALGEBRA

PROGRAM FIVE

USING POSITIVE EXPONENTS

A. SYNOPSIS

This program will:

Define exponents using positive integers and scientific notation.

Illustrate the usefulness of exponents. Discover the rules of using

positive exponents.

Reinforce the use of exponents in computing square and cubic measures.

NCTM STANDARDS ADDRESSED IN PROGRAM FIVE

STANDARD	GRADES 6 – 8, 9 – 12
Number and Operations	Students should understand numbers, ways of representing numbers, relationships among numbers and number systems
Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols
Geometry	Students should use visualization, spatial reasoning, and geometric modeling to solve problems.

B. VOCABULARY

VARIABLE........ a symbol, usually a letter, used to represent any element of a given set of more than one element; a placeholder for a number.

FACTOR .... when two or more numbers or **variables** are multiplied to form a product, each number or **variable** is a **factor** of the product.

BASE.................. the term to be used as a **factor;** the term raised to an **exponent.**

EXPONENT .... small raised number that tells how many times a **base** is to be used as a **factor** (or how many times the **base** is to be multiplied by itself).

POWER .... product in which the **factors** are identical; a number which is expressed by means of a **base** and an **exponent.**

SCIENTIFIC
NOTATION........ a system of using **exponents** to represent very large numbers.

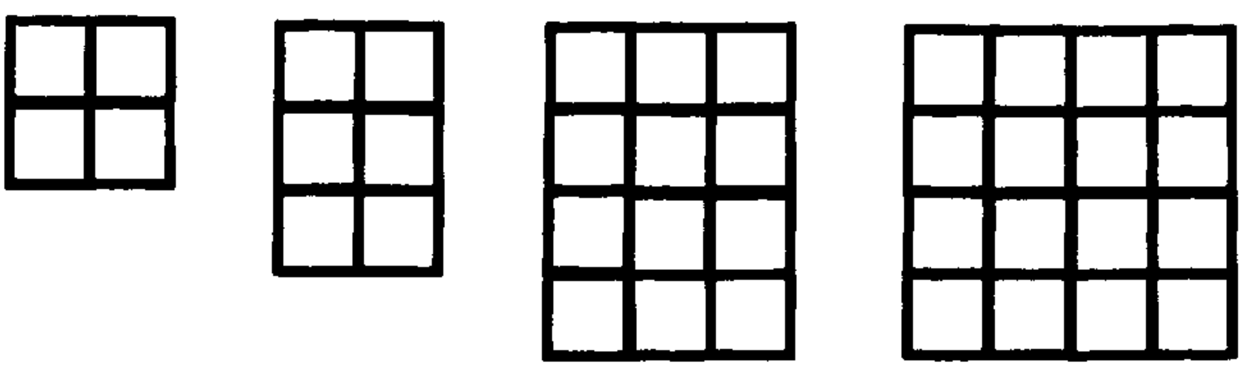
C. PRE-PROGRAM ACTIVITY

1. Give students 1 cm. grid paper. Have them draw a square with sides of **2**. Write on the board $\mathbf{2^2}$. Have them count the number of blocks within the blocks. Show them that $\mathbf{2^2 = 2 \bullet 2 = 4}$. Continue with $\mathbf{3^2}$, $\mathbf{4^2}$, etc.

   Display grids such as these. Ask the following questions:

   a) Do the following grids represent square numbers?

   b) If the answer is yes, what number is squared?



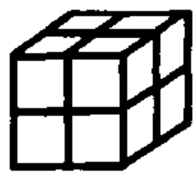
1) ______   2) ______   3) ______   4) ______

C. PRE-PROGRAM ACTIVITY (Continued)

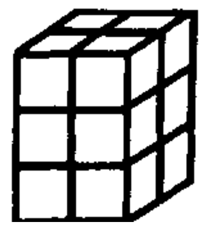
2. To show cubic numbers, give the students connecting blocks. Ask them to make a cube that is **2** blocks long, **2** blocks wide, and **2** blocks tall. Ask the students to count the number of blocks in the cube that they have made. Show them that $\mathbf{2^3 = 2 \bullet 2 \bullet 2 = 8}$. Continue with building $\mathbf{3^3}$ **and** 4^3.

   Show figures such as the following on the overhead or blackboard. Ask which show cubic numbers and which do not. If the figure shows a cubic number, what is the number?

1) ________



2) ________



3) ________

4) ________

D. DISCUSSION QUESTIONS

1. Why are exponents useful? Can you think of any real applications or examples of exponents?

2. A certain colony of bacteria doubles in population every day. Name at least two ways to determine the population of bacteria at the end of the seventh day?

3. If 4^2 can be represented by a square with the length and width of 4, and 4^3 can be represented by a cube with the length, width, and height of 4, is there a way to physically represent 4^4 ? Explain.

4. Is 2^3 equal to 3^2 ? Explain.

E. FOLLOW-UP ACTIVITIES

1. Paper Folding

Have students fold a piece of paper in half one time. Write $\mathbf{2^1}$ on the board. Tell them that this means that you have folded the paper into **2 parts 1** time. How many parts do they have? $\mathbf{2^1 = 2}$. Have them fold the paper in half a second time. Write $\mathbf{2^2}$ and ask them to count the parts. $\mathbf{2^2 = 4}$. Continue folding a third and fourth time showing that $\mathbf{2^3 = 8}$ and $\mathbf{2^4 = 16}$. Do the same process with folding paper into thirds.

2. Exponential Chart

Give students a chart showing the patterns in exponential numbers.

Exponent Form	Read	Number
		100,000
10^4		
		1,000
	Ten to the second power, or ten squared	
10^1	Ten to the first power	

3. Practice for Exponents

Write in exponential form:

a) 5 • 5 _____

b) 4 • 4 • 4 _____

c) 1 • 1 • 1 • 1 _____

Write the number:

a) 2^3 _____

b) 3^2 _____

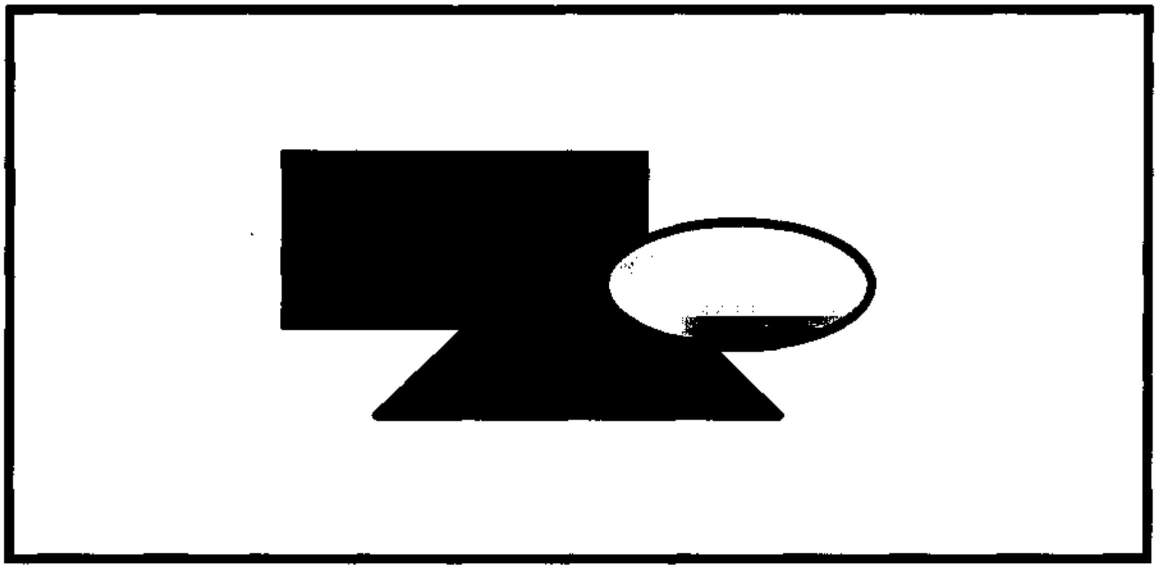
c) 1^5 _____

d) 6^1 _____

E. FOLLOW-UP ACTIVITIES (Continued)

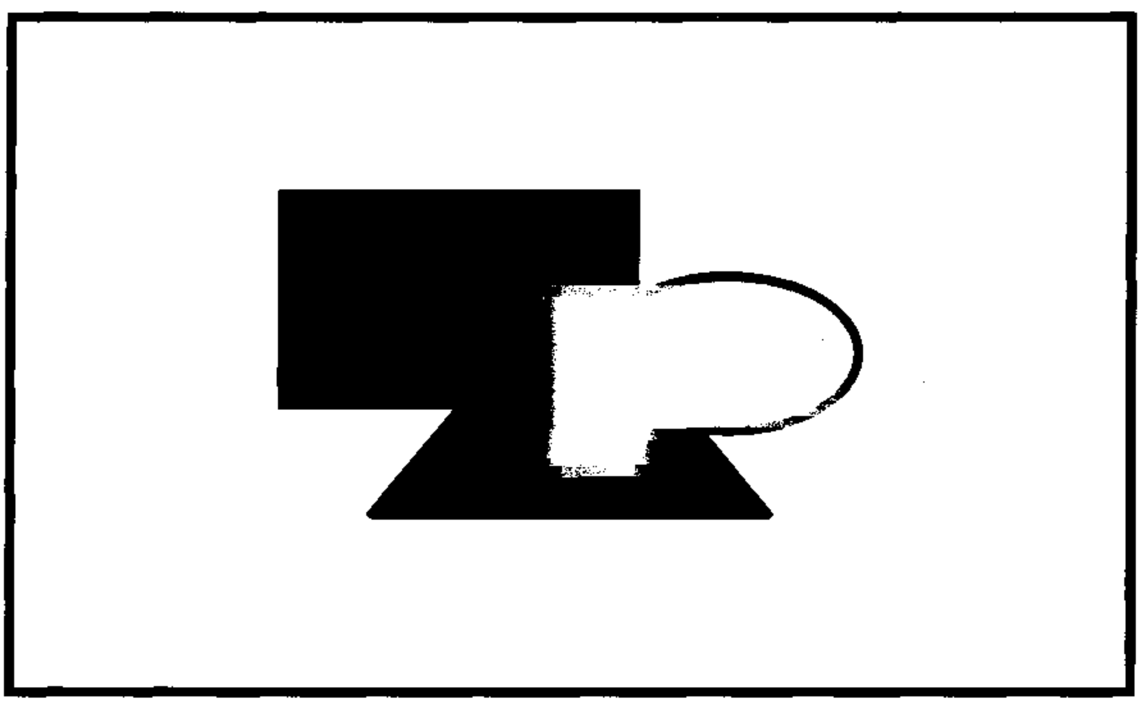
4. Practice for Scientific Notation

   Complete the problem, writing the numbers in scientific notation.



Write an expression in scientific notation that represents the following:

The Earth is approximately 93 million miles from the Sun.



E. **FOLLOW-UP ACTIVITIES** (Continued)

6. Multiplying and Dividing with Bases Other Than 10

   Have students work in groups to answer the following:

   MULTIPLICATION

   What is the meaning of 3^2 ? What is the meaning of 3^1?

   What is the meaning of $3^2 \bullet 3^1$?

   Can you determine a shortcut for multiplying powers with like bases? Give several examples to support your method. Will your shortcut work if the bases are not the same?
   Explain.

   Can you apply your shortcut to variables used as bases?

   $$x^3 \bullet x^{10} =$$

   DIVISION

   Develop a shortcut for division in the same way. Be sure to give several examples to support your method.

   $$\frac{5^6}{5^2}$$

F. STUDENT WORKSHEET

USING POSITIVE EXPONENTS **Name** ______________________

A. Write in exponential form.

1. $3 \bullet 3 \bullet 3 \bullet 3$
2. $(-2) \bullet (-2) \bullet (-2)$
3. $x \bullet x$
4. $y \bullet y \bullet y \bullet y \bullet y$

B. Write the number.

1. 2^4
2. $(-1)^3$
3. 0^5
4. 256^1

C. Write the following in scientific notation.

1. **635,000,000**
2. **The value in dollars of sixty thousand nickels.**
3. **The number of seconds in one day.**

D. Multiply the following.

1. $5^3 \bullet 5^4$
2. $x^3 \bullet x^7$

Program Five Using Positive Exponents

Student Worksheet F

A. 1) 3^4 2) $(-2)^3$ 3) x^2 4) y^5

B. 1) 16   2) -1   3) 0   4) 256

C. 1) $6.35 X 10^8$ 2) $3.0 X 10^3$ 3) $8.64 X 10^4$

D. 1) $5^7 = 78125$ 2) x^{10}

THE POWER OF ALGEBRA

PROGRAM SIX

POLYNOMIALS AND EQUATIONS

A. SYNOPSIS

This program will:

Develop an understanding of like terms and unlike terms.

Develop a definition of polynomials.

Explore the nature of different kinds of polynomials.

Present methods for solving two equations in two unknowns.

Demonstrate ways to solve verbal problems with two unknown quantities.

NCTM STANDARDS ADDRESSED IN PROGRAM SIX

STANDARD	GRADES 6 – 8, 9 –12
Algebra	Students should understand patterns, relations, and functions.
Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols.

B. VOCABULARY

CONSTANT ... a known quantity; a number.

FACTOR......... if two numbers a and b are multiplied together to form a product ab, then a and b are called the **factors** of the product **ab.** COEFFICIENT.. in the expression **5xy,** each factor is the **coefficient** of the other factors:

5 is the **coefficient of** xy

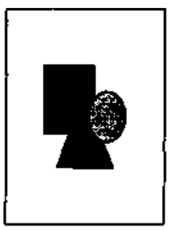
5x is the **coefficient of y**

5y is the **coefficient of x**

xy is the **coefficient of** 5 TERM .... a mathematical expression using numerals, or variables, or both, to indicate a product

is called a **term. Terms** are joined by plus (+) or minus (-) signs.

Example: **4x,** 7, **2xy,** and $\mathbf{\frac{x}{x+y}}$ are all **terms.**



NOTE TO THE TEACHER: "MONO" means ONE.
"BI" means TWO.
"TRI" means THREE.
"POLY" means MANY.

MONOMIAL......... a monomial is a term which is either a numeral, such as 5; or a variable, such as x; or a product of a numeral and one or more variables, such as $2x^3$ or -4xy.

BINOMIAL .... an expression consisting of exactly two unlike terms, such as $x^3 - y^3$.

TRINOMIAL .... an expression consisting of exactly three unlike terms.

Example: $ax^2 + bx + c$

POLYNOMIAL .... any sum of monomials.

Example: $5x^2 + 3y^2 - 3x + 2y$

LIKE TERMS.........terms of an algebraic expression with exactly the same variable part are called **like terms.** Example: **3a and 6a are like terms.**

UNLIKE TERMS ......terms that do not have exactly the same variable are **unlike terms.** Example: **3a and 6a^2 are unlike terms.**

C. PRE-PROGRAM ACTIVITY

1. Discuss what happens if you have 3 **apples and** 2 **oranges** and you put them all in the same bowl. You have a bowl containing 5 **pieces of fruit,** but they are still distinguishable as **3 apples and 2 oranges.** The same thing is true if you have 3x and 2y. If you wish to add them, all you can do is write 3x + 2y.

D. DISCUSSION QUESTIONS

1. Summarize the main ideas of the program.

2. Can you find any real life applications for two equations in two variables?

3. What happens to the solution if you have just one equation in two variables? Example: x + y = 6 (Infinite number of solutions.)

4. Graph x + y = 6 to see why it is said to have an infinite number of solutions.

5. Is $3x^2 + 4x - 6$ a polynomial? Explain. Are $3x^2$ and 4x like terms? Explain.

NOTE TO THE TEACHER: We call this a polynomial in one variable. Any equation in the form $ax^2 + bx + c = 0$ is called a polynomial equation in the second degree and also a quadratic equation.

E. FOLLOW-UP ACTIVITIES

1. Can you solve $2x + 3y = 5$ and $4x + 6y = 10$? Graph them. These are called **dependent equations.**

2. Can you solve $2x + 3y = 5$ and $2x + 3y = 10$? Graph them. These are **parallel lines.**

3. Solve $2x + y = 8$ and $y - x = 2$. Graph $2x + y = 8$ and $y - x = 2$. What are the coordinates of the point when the two lines intersect?

4. In the set of equations, is $y = 8 - 2x$ the same as $2x + y = 8$? By using substitution in the second equation, then, is $(8 - 2x) - x = 2$ true? Solve this equation, and compare the solution to the solutions in Question 3, above.

5. Discuss the three different ways to find the solution to two equations with two variables. The first method is called the **algebraic method;** the second, the **graphing method;** and the third, the **substitution method.**

6. Which method(s) are most reliable for getting the solution set? Which are least reliable? Explain.

7. Have students work in groups to set up and solve reading problems that require two unknowns.

Example: If a boat that requires 3 **hours** to travel **60 miles downstream** needs 5 **hours to return,** find the **rate** the boat can travel in still water, and the **rate** of the current.

Remember: $d = rt$

Solution: Let **r = rate of the boat in still water c = rate of the current**

	RATE MPH	TIME HOURS	DISTANCE 60 MILES
With the current:	**r+c**	**3**	**3(r + c)**
Against the current:	**r-c**	**5**	**5(r - c)**

Thus, $3(r + c) = 60$
$5(r - c) = 60$

By the distributive property:

$$3r + 3c = 60$$
$$5r - 5c = 60$$

Multiply both members of the first equation by 5:

$$15r + 15c = 300$$

Multiply both members of the second equation by 3:

$$15r + 15c = 300$$
$$\underline{15r - 15c = 180}$$

$$30r = 480$$
$$r = 16$$

$$3(16) + 3c = 60$$
$$48 + 3c = 60$$
$$3c = 12$$
$$c = 4$$

Check: $3(16 + 4) = 60$ $5(16 - 4) = 60$

$3(20) = 60$ $5(12) = 60$

$60 = 60$ $60 = 60$

F. STUDENT WORKSHEET

POLYNOMIALS AND EQUATIONS **Name**__________________________

A. Simplify:

1. $15x + 3y + 20x + (-y)$

2. $2(-2x^2 - 4y) + [- (-3x^2 + 17y)]$

3. $-3 [2(-2 + 4xy) - 3] + 4(-xy + 5)$

B. Simplify and solve:

1. $(x - 1) + (x - 2) + x = 0$

2. $2(x - 3) - 3x = -5$

3. $3x + 2[5(1 + x) - 2(1 - x)] = 40$

C. Solve the system for both variables using the algebraic method.

1. $x + y = 5$
   $x - y = 3$

2. $a + 2b = 1$
   $a - b = 4$

3. $5x - 2y = 1$
   $3x - 7y = -18$

F. **STUDENT WORKSHEET (Page 2)**

POLYNOMIALS AND EQUATIONS **Name**______________________

D. Solve by graphing:

1. $x = y + 1$, $y = x + 1$
2. $2x + 7y = 3$, $x - 3y = 5$
3. $y = 2x + 2$, $y = 2x - 3$
4. $2x - y = 2$, $6x - 3y = 6$

E. Solve by using substitution:

1. $3x - y = 5$, $5x - 2y = 8$
2. $x - 2y = 0$, $4x - 3y = 15$
3. $12r + 3s = 51$, $7r - 6s = 22$

F. Solve each equation by using two variables:

1. **If a boat can travel 18 miles downstream in 2 hours, but needs 6 hours to get back to the starting point, what is the rate of the boat in still water, and what is the rate of the current.**

2. **A scuba dive shop offers two trip packages. Package A is one night with two dives for $187. Package B is two nights and three dives for $320. What would be the cost of a single night and the cost of a single dive?**

Program Six Polynomials and Equations

Student Worksheet F

A. 1) $35x + 2y$ 2) $-x^2 - 25y$ 3) $-28xy + 41$

B. 1) $x = 1$ 2) $x = -1$ 3) $x = 2$

C. 1) $x = 4, y = 1$ (4,1) 2) $a = 3, b = -1$ (3,-1) 3) $x = 43/29, y = 93/29$ $(\frac{43}{29}, \frac{93}{29})$

D. 1) Since the lines do not intersect, there is no solution.

2) (2, -1)

3) Since the lines are parallel and do not intersect, there is no solution.

4) All points on the line are the solution, since the lines lie on top of each other.

E. 1) (2,1) 2) (6,3) 3) (4,1)

F. 1) rate of boat is 6 miles per hour, rate of current is 3 miles per hour

2) one dive costs $54 and one night costs $79

THE POWER OF ALGEBRA

PROGRAM SEVEN

FACTORING I

A. SYNOPSIS

This program will:

Define variable, constant, term, polynomial, monomial, binomial, trinomial, factor, multiple, prime factor, divisor, the degree of a polynomial, and quadratic equation.

Develop some techniques for factoring polynomials of the form $x^2 + bx + c$.

Determine solutions for the equation $x^2 + bx + c = 0$.

NCTM STANDARDS ADDRESSED IN PROGRAM SEVEN

STANDARD	GRADES 6 – 8, 9 – 12
Number and Operations	Students should understand numbers, ways of representing numbers, relationship among numbers, and number systems.
Algebra	Students should understand patterns, relations, and functions.
Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols.

B. VOCABULARY

VARIABLE .... a letter of the alphabet used to stand for an unknown quantity.

CONSTANT .... a known quantity; a number.
Example: 6 ,140 , -8 , 3 / 5 , 2.7

TERM .... a mathematical expression using numerals, or variables, or both, to indicate a product is called a **term. Terms** are joined by plus (+) or minus (-) signs. Example: $4x$, 7 , $2xy$, and $x / (x + y)$ are all **terms.**

NOTE TO THE TEACHER: In this program, constants will be integers.

MONOMIAL .... a term which is either a numeral, such as 5; a variable, such as x; or a product of a numeral and one or more variables.
Example: $\mathbf{-3v^2}$, $2t$, $4x^3$, 5 , $\mathbf{2ab^2}$

BINOMIAL .... an expression consisting of exactly two unlike terms.
Example: $2x - 5$, $y^2 - y$, $a - b$

TRINOMIAL .... an expression consisting of exactly three unlike terms.
Example: $x^2 - 5x + 6$,$3y^2 + 10y + 8$, $a + b - c$

POLYNOMIAL ...any sum of monomials.
Example: $2x - 5$, $3x^2 + 6x - 8$, $y^2 + 5y + 6$, $a^2b + 2ab + c$

FACTOR .... if two numbers a and b are multiplied together to form a product ab, then a and b are called the **factors** of the product ab.
Example: 6 and x are factors of $6x$
2 and $(x - 5)$ are factors of $2(x - 5)$
$(x - 2)$ and $(x + 3)$ are factors of $(x - 2)(x + 3)$.

NOTE TO TEACHER
a) If a and b are the factors of ab, then ab is said to be a multiple of a and a multiple of b.
b) a is called a prime factor of the product ab if the only divisors of a are 1, - 1, a, and -a.

If p(x) is a POLYNOMIAL, then q(x) and h(x) are called FACTORS or DIVISORS of p(x) if $p(x) = q(x) \bullet h(x)$.

Example: $x - 3$ is a factor of $x^2 - 5x + 6$ since $x^2 - 5x + 6 = (x - 2) \bullet (x - 3)$.

q(x) is called a PRIME FACTOR of p(x) if $p(x) = q(x) \bullet h(x)$, and if the only FACTORS of q(x) are q(x), -q(x), 1 , and -1. In the example above, $x - 3$ is a PRIME FACTOR of $x^2 - 5x + 6$.

The DEGREE of a MONOMIAL is the sum of the EXPONENTS of the VARIABLES. Example:

xy^2 is a MONOMIAL of DEGREE THREE. a is a MONOMIAL of DEGREE ONE. 5 is a MONOMIAL of DEGREE ZERO.

The DEGREE of a POLYNOMIAL is the highest degree of the monomials of which it is composed.

Example: $x - 5$ is a POLYNOMIAL of DEGREE ONE.
$2x^2 - 5x + 11$ is a POLYNOMIAL of DEGREE TWO. $3x^2y + 2xy + 5$ is a POLYNOMIAL of DEGREE THREE.

A polynomial of second degree or degree two is called a QUADRATIC POLYNOMIAL.

Example: $3x^2 - 12x + 18$ $x^2 - 16$

An equation in one variable, with that variable appearing in degree two, but no greater degree, is called a QUADRATIC EQUATION.

Example: $x^2 + 8x + 16 = 0$ $x^2 - 25 = 0$

C. FOLLOW-UP ACTIVITIES

1. Using the **distributive property of multiplication with respect to addition,** show that $(x - 5) \cdot (x + 7) = x^2 + 2x - 35$. What other property of the real numbers must be used to show this?

2. If $a \cdot b = 0$, what does this tell us about either a or b? Is this also true for $x - 2$ and $x + 7$ if $(x-2)(x+7) = 0$?

There are various methods for factoring different kinds of polynomials, but the method used to factor polynomials of the form $x^2 + bx + c$ is referred to as "factoring by trial and error, or factoring by observation." For example, which of the following is the correct factorization of the trinomial $x^2 - 8x + 16$ into a product of two binomials?

$$\mathbf{x^2 - 8x + 16 = (x + 4)(x + 4)}$$
$$\mathbf{x^2 - 8x + 16 = (x + 8)(x + 2)}$$
$$\mathbf{x^2 - 8x + 16 = (x - 8)(x - 2)}$$
$$\mathbf{x^2 - 8x + 16 = (x + l)(x + 16)}$$
$$\mathbf{x^2 - 8x + 16 = (x - 1)(x - 16)}$$
$$\mathbf{x^2 - 8x + 16 = (x - 4)(x - 4)}$$

Obviously, the last choice is the correct factorization. Notice that the first term x^2 of the trinomial $x^2 - 8x + 16$ **is** the product of the first terms of the two binomials, and the last term **+16** of the trinomial is the product of the last terms of two binomials. The second term of the trinomial is the sum of two products: the product of the first term of the first binomial and the second term of the second binomial, and the product of the second term of the first binomial and the first term of the second binomial.

In this example, $x^2 - 8x + 16$ factors into $(x + p)(x + q)$ where

$$\mathbf{p \cdot q = 16} \text{ and}$$
$$\mathbf{px + qx = -8x}.$$

Therefore, to factor $x^2 - 8x + 16$ into $(x + p)(x + q)$, we must find p and q such that

$$p \bullet q = 16$$
$$p + q = -8$$

The possibilities for p and q such that $p \bullet q = 16$ are:

p	q
1	16
-1	-16
2	8
-2	-8
4	4
-4	-4

NOTE TO THE TEACHER: Many students have trouble writing down ALL of the factors of a number. A simple method is to begin with *1* ; in this example, 1 • 16 and (-1) (-16). Then try 2 : 2 . 8 and (-2) (-8). And so on, until you begin repeating. This will occur with the factors 4 • 4 and (-4) (-4).

However, only when p = -4 and q = -4 do we have p + q = -8. The possibilities for p and q may be narrowed by noting the signs of the trinomial. If the sign of the constant term is "+" then only two possibilities exist:

Either both p and q are positive, or

both p and q are negative.

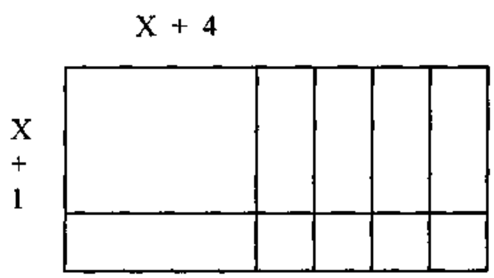
If the sign of the middle term in the polynomial is negative, and if the constant term is positive, then both p and q are negative. If the sign of the constant term is negative, then one of p or q is positive and the other negative.

In our example, $x^2 - 8x + 16$, the sign of the constant sign is "+" and the sign of the middle term is "-", so our chart of values for p and q can be reduced to the following:

p	q
-1	-16
-2	-8
-4	-4

4. Use algebra tiles to model factoring as the process of translating the product of area into the factors of length and width.

Example: $X^2 + 5X + 4 = (X + 4) \bullet (X + 1)$



D. DISCUSSION QUESTIONS

1. Show how the technique above can be used to factor the difference of two perfect squares as $\mathbf{x^2 - 81}$.

2. Show how the distributive property of multiplication with respect to addition may be used to factor $\mathbf{6x^2 + 36x - 96}$.

3. Students should work in groups to discuss and illustrate how they could use factoring to find the solutions to the following equations:

$$\mathbf{x^2 - 81 = 0}$$

$$\mathbf{6x^2 + 36x - 96 = 0}$$

4. Explain how you would know whether the solutions, $\mathbf{x = 5}$ and $\mathbf{x = -2}$, are correct for the quadratic equation, $\mathbf{x^2 - 3x = 10}$.

E. **STUDENT WORKSHEET**

FACTORING I **Name_______________________**

Factor the following:

1. $x^2 - 5x + 6$

2. $t^2 + 11t + 18$

3. $v^2 - 64$

4. $bx^2 - 25b$

5. $64 - 16y^2$

6. $36 + x^2 - 13x$

7. $y^2 + 10y + 25$

8. $x^2 + 2x - 35$

9. $x^2 - 2x - 35$

10. $t^2 + 11t + 30$

11. $x^2 - 15x$

E. **STUDENT WORKSHEET** (Page 2)

FACTORING I **Name** ____________________________

Solve the following quadratic equations. Be sure to check your answers using substitution.

1. $x^2 - 3x - 18 = 0$

2. $y^2 - 3y = 0$

3. $x^2 - 13x + 36 = 0$

4. $t^2 - 25 = 0$

5. $v^2 + 16v + 64 = 0$

6. $5y^2 + 10y - 240 = 0$

7. $a^2 + 2a = 24$

8. $c^2 + 48 = 19c$

9. $50 + 5x = x^2$

10. $-y^2 + y = -90$

Program Seven Factoring I

Student Worksheet E

1) $(x - 2)(x - 3)$

2) $(t + 9)(t + 2)$

3) $(v - 8)(v + 8)$

4) $b(x - 5)(x + 5)$

5) $16(2 + y)(2 - y)$

6) $(x - 9)(x - 4)$

7) $(y + 5)(y + 5) = (y + 5)^2$

8) $(x + 7)(x - 5)$

9) $(x - 7)(x + 5)$

10) $(t + 5)(t + 6)$

11) $x(x - 15)$

Page 2

1) $x = -3$ or $x = 6$

2) $y = 0$ or $y = 3$

3) $x = 9$ or $x = 4$

4) $t = -5$ or $t = 5$

5) $v = 8$

6) $y = -8$ or $y = 6$

7) $a = 4$ or $a = -6$

8) $c = 16$ or $c = 3$

9) $x = 10$ or $x = -5$

10) $y = 10$ or $y = -9$

THE POWER OF ALGEBRA

PROGRAM EIGHT

FACTORING II

A. SYNOPSIS

This program will:

Review the vocabulary from Program Seven.

Develop techniques for factoring the quadratic $ax^2 + bx + c = 0$.

NCTM STANDARDS ADDRESSED IN PROGRAM EIGHT

STANDARD	GRADES 6 – 8, 9 – 12
Number and Operations	Students should understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Algebra	Students should understand patterns, relations, and functions.
Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols.

B. VOCABULARY

VARIABLE .... a letter of the alphabet used to stand for an unknown quantity.

CONSTANT .... a known quantity; a number.
Example: **6 , 140 , -8 , 3/5 , 2.7**

TERM .... a mathematical expression using numerals, or variables, or both, to indicate a product is called a **term.**
Terms are joined by plus (+) or minus (-) signs.
Example: **4x , 7 , 2xy** , and **[(x) / (x + y)]** are all **terms.**

NOTE TO THE TEACHER: In this program, constants will be integers.

MONOMIAL .... a term which is either a numeral, such as **5**; a variable, such as **x**; or a product of a numeral and one or more variables.
Example: $-3v^2$, $2t$, $4x^3$, 5 , $2ab^2$

BINOMIAL .... an expression consisting of exactly two unlike terms.
Example: $2x - 5$, $y^2 - y$, $a - b$

TRINOMIAL .... an expression consisting of exactly three unlike terms.
Example: $x^2 - 5x + 6$, $3y^2 + 10y + 8$, $a + b - c$

POLYNOMIAL .... any sum of monomials.
Example: $2x - 5$, $3x^2 + 6x - 8$, $y^2 + 5y + 6$, $a^2b + 2ab + c$

FACTOR if two numbers **a** and **b** are multiplied together to form a product **ab**, then **a** and **b** are called the **factors** of the product **ab**.
Example: **6** and **x** are factors of **6x**
2 and **(x - 5)** are factors of **2(x - 5)**
(x - 2) and **(x + 3)** are factors of **(x - 2) (x + 3)**

NOTE TO THE TEACHER:
a) If a and b are the factors of ab, then ab is said to be a multiple of a and a multiple of b.
b) a is called a prime factor of the product ab if the only divisors of a are 1, - 1, a, and -a.

If **p(x)** is a POLYNOMIAL, then **q(x)** and **h(x)** are called FACTORS or DIVISORS of **p(x)** if **p(x) = q(x) • h(x).**

Example: **x - 3** is a FACTOR of $x^2 - 5x + 6$ since
$x^2 - 5x + 6 = (x - 2) \bullet (x - 3)$.

q(x) is called a PRIME FACTOR of **p(x)** if **p(x) = q(x) • h(x)**, and if the only FACTORS of **q(x)** are **q(x), -q(x), 1,** and **- 1**. In the example above, **x - 3** is a PRIME FACTOR of $x^2 - 5x + 6$.

The DEGREE of a MONOMIAL is the sum of the EXPONENTS of the VARIABLES.

Example: xy^2 is a MONOMIAL of DEGREE THREE

a is a MONOMIAL of DEGREE ONE

5 is a MONOMIAL of DEGREE ZERO.

The DEGREE of a POLYNOMIAL is the highest degree of the monomials of which it is composed.

Example: **x - 5** is a POLYNOMIAL of DEGREE ONE.

$2x^2 - 5x + 11$ is a POLYNOMIAL of DEGREE TWO.

$3x^2y + 2xy + 5$ is a POLYNOMIAL of DEGREE THREE.

A polynomial of second degree or degree two is called a QUADRATIC POLYNOMIAL.

Example: $3x^2 - 12x + 18$

$x^2 - 16$

An equation in one variable, with that variable appearing in degree two, but no greater degree, is called a QUADRATIC EQUATION.

Example: $x^2 + 8x + 16 = 0$
$x^2 - 25 = 0$

The STANDARD FORM of a quadratic polynomial is $ax^2 + bx + c$, where **a** is the COEFFICIENT of x^2; **b** the COEFFICIENT of **x**; and **c** the constant term, or the constant coefficient.

C. FOLLOW-UP ACTIVITIES

1. In Program Seven, we considered only those trinomials having **1** as a coefficient of $\mathbf{x^2}$.
   We now consider the factorization of the trinomial $\mathbf{ax^2 + bx + c}$ where $\mathbf{a \neq 1}$.
   Thus, we wish to write:

$$\mathbf{ax^2 + bx + c = (mx + n)\,(px + q)}$$

For this to be a correct factorization of $\mathbf{ax^2 + bx + c}$, it is necessary that

$$\begin{aligned} \mathbf{mp} &= \mathbf{a} \\ \mathbf{nq} &= \mathbf{c} \\ \mathbf{mq + np} &= \mathbf{b} \end{aligned}$$

Notice that

mp is the product of the coefficients of the first terms **(F)** of the two factors;
mq is the product of the two outer **(O)** coefficients;
np is the product of two inner **(I)** coefficients; and
nq is the product of the two last **(L)** coefficients.

Thus, an easy device for remembering these simple equations is the word **F O I L**.

For example, we wish to find **m** , **n** , **p** , and **q** such that

$$\mathbf{2x^2 + 11x + 15 = (mx + n)\,(px + q)}$$

Thus, we must have

$$\overset{F}{\mathbf{mp}} \mathbf{= 2}$$

$$\overset{O}{\mathbf{mq}} \mathbf{+} \overset{I}{\mathbf{np}} \mathbf{= 11}$$

$$\overset{L}{\mathbf{nq}} \mathbf{= 15}$$

The possibilities for m, p, n, and q are as follows:

m	**p**
2	1
-2	-1
1	2
-1	-2

n	**q**
1	15
-1	-15
3	5
-3	-5
15	1
-15	-1
5	3
-5	-3

But since all the coefficients in the polynomial $\mathbf{2x^2 + 11x + 15}$ are positive, then only the positive values of **m** , **p** , **n** , and **q** need be considered. Thus, the only possible values of **m** , **p** , **n** , and **q** are:

m	**p**
1	2
2	1

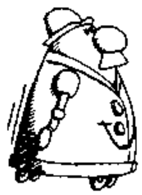
n	**q**
1	15
3	5
5	3
15	1

However, when $\mathbf{m} = 2$, $\mathbf{p} = 1$, $\mathbf{n} = 5$, and $\mathbf{q} = 3$, we have

$$\mathbf{mq} + \mathbf{np} = (2)(3) + (5)(1)\ 11\ .$$

Thus, $2x^2 + 11 + 15 = (2x + 5)(x + 3)$.

NOTE TO THE TEACHER: If the coefficient of x^2 is negative, use the distributive property of multiplication over addition to factor out (- 1) , and then proceed as illustrated.



NOTE TO THE TEACHER: Before attempting to factor a quadratic polynomial, it should always be written in the form $ax^2 + bx + c$. The quadratic is then said to be written in <u>descending powers</u> of the variable.

If the quadratic

$$ax^2 + bx + c$$

factors as

$$ax^2 + bx + c = (mx + n)\ (px + q),$$

then the equation

$$ax^2 + bx + c = (mx + n)\ (px + q) = 0$$

only if

$$mx + n = 0 \text{ or } px + q = 0$$

or both are equal to zero. Thus,

$$(mx + n)\ (px + q) = 0$$

when either

$$mx + n = 0 \quad \text{or} \quad px + q = 0;$$

$$mx = -n \quad \text{or} \quad px = -q;$$

$$x = -(n / m) \quad \text{or} \quad x = -\ (q / p)$$

and the solutions for

$$ax^2 + bx + c = 0$$

are

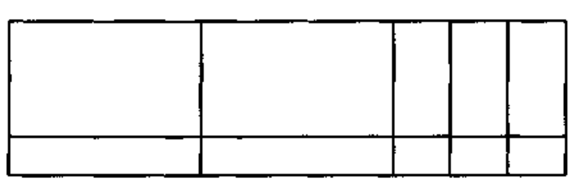
$$x = -(n / m) \quad \text{and} \quad x = -(q / p).$$

2. Use algebra tiles to model factoring as the process of translating the product of area into the factors of length and width.

Example: $2X^2 + 5X + 3 \quad = \quad (2X + 3) \bullet (X + 1)$

2X + 3

X
+
1



D. DISCUSSION QUESTIONS

1. Can the technique discussed in Section C be applied to factoring the quadratic

$$x^2 - 36 = 0\ ?$$

2. Can the technique discussed in Section C be applied to factoring the quadratic

$$\mathbf{3t^4 + 20t^2 + 32\ ?}$$

3. Students should work in groups to prove that if $ax^2 + bx + c$

factors as $ax^2 + bx + c = (mx + n)\ (px + q)$,

that $x = - m / n$ and $x = - q / p$

are solutions of the quadratic equation $ax^2 + bx + c = 0$.

D. DISCUSSION QUESTIONS

1. Can the technique discussed in Section C be applied to factoring the quadratic

$$x^2 - 36 = 0 \; ?$$

Can the technique discussed in Section C be applied to factoring the quadratic

$$\mathbf{3t^4 + 20t^z + 32 \; ?}$$

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that $x = - m / n$ and $x = - q / p$

are solutions of the quadratic equation $ax^2 + bx + c = 0$.

E. STUDENT WORKSHEET

FACTORING II Name_______________________

Factor the following:

1. $4x^2 + 12x + 9$

2. $6y^2 + 7y - 24$

3. $25x^2 - 45x - 36$

4. $5t^2 + 7t - 6$

5. $13y + 24 - 2y^2$

6. $12t^2 - 52t + 30$

7. $4x^2 + 10x + 24$

8. $-y - 35 + 6y^2$

9. $9x^2 - 81$

10. $2x^2 + 10x + 12$

11. $3x^2 - 15x$

12. $27y^2 - 108$

E. STUDENT WORKSHEET (Page 2)

FACTORING II Name ____________________________

Solve the following quadratic equations. Be sure to check your answers using substitution.

1. $3x^2 + 6x - 105 = 0$
2. $4y^2 + 3y - 1 = 0$
3. $2x(2x + 6) + 9 = 0$
4. $2x^2 + 18x - 72 = 0$
5. $27v^2 - 108 = 0$
6. $13y + 24 - 2y^2 = 0$
7. $3 = 2x^2 + x$
8. $12 - 17c = -6c^2$
9. $9x^2 - 22x + 8 = 0$
10. $-30y + 40 = -5y^2$

Program Eight Factoring II

1) $(2x + 3)(2x + 3) = (2x + 3)^2$

2) $(2y - 3)(3y + 8)$

3) $(5x + 3)(5x - 12)$

4) $(5t - 3)(t + 2)$

5) $(2y + 3)(-y + 8)$

6) $2(6t^2 - 26t + 15)$

7) $2(2x^2 + 5x + 12)$

8) $(2y - 5)(3y + 7)$

9) $9(x - 3)(x + 3)$

10) $2(x + 2)(x + 3)$

11) $3x(x - 5)$

12) $27(y + 2)(y - 2)$

Page 2

1) $x = -7$ or $x = 5$

2) $y = 1/4$ or $y = -1$

3) $x = -3/2$

4) $x = 3$ or $x = -12$

5) $x = 2$ or $x = -2$

6) $y = -3/2$ or $y = 8$

7) $x = -3/2$ or $x = 1$

8) $c = 3/2$ or $c = 4/3$

9) $x = 4/9$ or $x = 2$

10) $y = 4$ or $y = 2$

THE POWER OF ALGEBRA

PROGRAM NINE

FRACTIONS

A. SYNOPSIS

This program will:

show the relationship between arithmetic fractions and algebraic fractions.

reduce algebraic fractions by dividing out common factors, add,

subtract, multiply, and divide algebraic fractions, solve

equations involving algebraic fractions.

NCTM STANDARDS ADDRESSED IN PROGRAM NINE

STANDARD	GRADES 6 - 8
Number and Operation	Students should understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols.

B. VOCABULARY

FACTOR.............. when two or more numbers or variables are multiplied to form a product, each number or variable is a **factor** of the product.

TERM.................. a mathematical expression using numerals, or variables, or both, to indicate a product is called a **term. Terms** are joined by plus (+) or minus (-) signs. Example: **4x , 7 , 2xy** , and **[(x) / (x + y)]** are all **terms.**

COMMON
FACTOR.............. a number, variable, or combination of numbers and variables which is a factor of each of two or more terms.

NUMERATOR........the top number of a fraction.

DENOMINATOR.... the bottom number of a fraction.

RECIPROCAL......... for all non-zero numbers **a**, the **reciprocal** is the unique real number **(1 / a)**, such that **a • (1 / a) = 1** and **(1 / a) • a = 1**.

LEAST
COMMON
MULTIPLE........... the smallest number, variable, or combination of numbers and variables which is a multiple of all of the given terms.

LEAST
COMMON
DENOMINATOR.... least common multiple of the denominator of two or more fractions.

SIMPLIFIED
FORM.................. if the numerator and denominator of a rational expression have no common factors except **1** and **–1**.

C. PRE-PROGRAM ACTIVITY

Students should work in groups to review the four basic operations on fractions and the following terminology: **numerator, denominator, term, factor, common factor, simplest form, least common multiple, lowest common denominator, and reciprocal.** Each group should illustrate these terms and operations in the following problems:

1. **(1 / 2) + (2 / 3) =**

2. **(1 / 2) – (2 / 5) =**

3. **(1 / 2) • (2 / 3) =**

4. **(3 / 4) ÷ (3 / 8) =**

Explain that today the students will see a program that does these very operations on problems that include variables. Tell them that the rules are similar to the ones they followed to complete the pre-program activity.

D. DISCUSSION QUESTIONS

1. How would you compare operations with numerical fractions to operations with algebraic fractions?

2. Why is it important to recognize when a variable is in the denominator?

3. Why can you reduce [(x +1) / (x +1)] to 1, but not reduce [(x) / (x +1)] to 1?

4. Identify and explain the mistake made in these problems.

(a / 3) + (b / 6) = [(a + b) / (9)]

$(a^2 / b^2) \bullet (b^3 / a^6) = (a^8 / b^5)$

(a / b) ÷ (c / d) = (bc) / (ad)

[(x – 1) / (x)] = 1

E. PROBLEMS FROM THE PROGRAM

1. Reduce: $\frac{28}{49} = \frac{4}{7}$

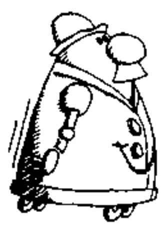
2. Reduce: $\frac{6x^2y}{9xy} = \frac{2x}{3}$

NOTE TO THE TEACHER: Students are told to find the common factor and divide both the numerator and denominator by it. The actual factor process is not shown. You may want to show it. It helps to prepare the students for the third example.

$$\frac{28}{49} = \frac{7 \bullet 4}{7 \bullet 7} \text{ or } \frac{7 \bullet 2 \bullet 2}{7 \bullet 7}$$

$$\frac{6x^2y}{9xy} = \frac{2 \bullet 3x \bullet xy}{3 \bullet 3x \bullet y}$$

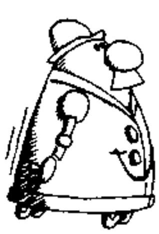
3. Simplify: $\frac{(x-1)^2}{x^2 - 9x + 8} = \frac{(x-1)(x-1)}{(x-8)(x-1)} = \frac{x-1}{x-8}$



NOTE TO THE TEACHER: In this problem, the factors must be shown in order for the students to see the common factors.

4. Multiply: $(3 / 8) \bullet (5 / 7) = (15 / 56)$

5. Multiply: $(a / b) \bullet (c / d) = (ac) / (bd)$



NOTE TO THE TEACHER: The similarity of the processes in the two problems should be shown. You may also want to include problems which require reducing or canceling.

6. Divide: **(3 / 8) ÷ (5 / 7) = (3 / 8) • (7 / 5) = (21 / 40)**

7. Divide: (a / b) ÷ **(c / d) = (a / b) • (d / c) = (ad / bc)**

NOTE TO THE TEACHER: The similarity in the processes in the two problems above should be shown to the students. The program states that, to solve the division problems, the divisors must be replaced by their reciprocals. You may want to include problems that require reducing or canceling.

8. Add: **(2 / 9) + (5 / 9) = [(2 + 5) / (9)] = (7 / 9)**

9. Add: **(a / c) + (b / c) = [(a + b) / (c)]**

10. Subtract: **(7 / 9) – (5 / 9) = [(7 – 5) / (9)] = (2 / 9)**

11. Subtract: **(a / c) – (b / c) = [(a – b) / (c)]**

NOTE TO THE TEACHER: When adding and subtracting fractions with like denominators, students simply need to add the numerators and place over the denominators. The second step in Problems 8 and 10 helps eliminate the problem of adding or subtracting the denominator. Students should be encouraged to write the problems horizontally rather than vertically in preparation for more complex algebraic fractions.

12. Add: **$(3 / 7) + (2 / 5) = [(15 + 14) / (35)] = (29 / 35)$**

13. Add: **$(5 / 3x) + (9 / 4x) = [(20 + 27) / (12x)] = (47 / 12x)$**

NOTE TO THE TEACHER: In each of the above two problems, students must find common denominators to add the problems. The least common denominator will be the smallest multiple that both denominators will divide into evenly. Do not waste time with large or difficult denominators. The program is more concerned with the process.

14. How long will it take to fill a pool with a garden hose alone if it takes five hours to fill it with a fill pipe alone and three hours to fill it with both a fill pipe and a garden hose?

$$w = r \bullet t$$

$$1 = [(1 / 5) + (1 / x)] \bullet 3$$

$$1 = (3 / 5) + (3 / x)$$

$$5x = 3x + 15$$

$$2x = 15$$

$$x = 7.5 \text{ hours}$$

NOTE TO THE TEACHER: You may need to review the process of clearing the denominators in an equation. To clear the denominators, multiply each term by the lowest common denominator of all of the terms.

15. How fast should Morianty's canoe go to get from the top of the falls to his barrel and back if he needs to complete the trip in five hours? His barrel is located 40km upstream from the top of the falls and he is facing a current of 6km/hr.

$$t = (d / r)$$

$$\text{downstream } t = [(40) / (x - 6)] \text{ going against the current}$$

$$\text{upstream } t = [(40) / (x + 6)] \text{ going with the current}$$

$$t = \text{downstream} + \text{upstream}$$

$$5 = [(40) / (x - 6)] + [(40) / (x + 6)]$$

then

$$LCD = (x - 6)(x + 6)$$

$$5(x - 6)(x + 6) = 40(x + 6) + 40(x - 6)$$

$$5(x^2 - 36) = 40(x + 6) + 40(x - 6)$$

$$5x^2 - 180 = 40x + 240 + 40x - 240$$

$$5x^2\ 180 = 80x$$

$$5x^2 - 80x - 180 = 0$$

$$5(x^2 - 16x - 36) = 0$$

$$5(x - 18)(x + 2) = 0$$

$$x = 18 \quad x = -2$$

Moriarity's speed can not equal -2 km/hr. So his canoe speed should be 18 km/hr.

F. STUDENT WORKSHEET

FRACTIONS **Name__________________**

A. Write in the simplest form:

1. $\dfrac{45}{70}$

2. $\dfrac{25x^3y^2}{5xy^2}$

3. $\dfrac{x^2 - 5x + 6}{x^2 - 9}$

B. Multiply:

1. $\dfrac{4}{5} \bullet \dfrac{15}{18}$

2. $\dfrac{x^2}{y} \bullet \dfrac{y^3}{x}$

C. Divide:

1. $\dfrac{2}{3} \div \dfrac{4}{9}$

2. $\dfrac{a^3}{c^2} \div \dfrac{b^2}{c^3}$

F. STUDENT WORKSHEET (Page 2)

Name________________

FRACTIONS

D. Add or subtract:

1. **(3 / 5) + (1 / 5)**
2. **(x / 3) + (y / 3)**
3. **(4 / n) + (6 / n)**
4. **(4 / 9) – (2 / 3)**
5. **(3 / 2x) – (x / 6x)**
6. **(1 / 3) – (2 / x)**
7. **[(2) + (x + 1)] + [(3) / (x – 1)]**

E. Solve the following equations:

1. **(1 / 4)x – (2 / 3) = 1**
2. **2[(1 / 3) + (1 / c)] = 1**

Program Nine   Fractions

Student Worksheet

A. 1) $\frac{9}{14}$ 2) $5x^2$ 3) $\frac{x-2}{x+3}$

B. 1) $\frac{2}{3}$ 2) $x\,y^2$

C. 1) $\frac{3}{2}$ 2) $\frac{a^3c}{b^2}$

Page 2

D. 1) $\frac{4}{5}$ 2) $\frac{x+y}{3}$ 3) $\frac{10}{n}$ 4) $-\frac{2}{9}$ 5) $\frac{9-x}{6x}$ 6) $\frac{x-6}{3x}$ 7) $\frac{x^2-2x}{(x-1)}$

E. 1) $6\frac{2}{3}$ 2) 6

THE POWER OF ALGEBRA

PROGRAM TEN

WORDS INTO SYMBOLS

A. SYNOPSIS

Explain how expressions or sentences in words can be translated into algebraic expressions or sentences.

Introduce several problem-solving strategies.

NCTM STANDARDS ADDRESSED IN PROGRAM TEN

STANDARD	GRADES 6 – 8, 9 –12
Algebra	Students should represent and analyze mathematical situations and structures using algebraic symbols.

B. VOCABULARY

SUM .... the result of adding two quantities.
DIFFERENCE............. the result of subtracting one quantity from another quantity.
PRODUCT .... the result of multiplying two quantities.
QUOTIENT................. the result of dividing one quantity by another quantity.
PERIMETER................ the distance around a geometric figure.
AREA........................... the measurement of the surface of a figure.

C. FOLLOW-UP ACTIVITIES

Many English words or phrases may be translated directly in algebraic symbols. Have students work in groups to come up with their own expressions for the symbols below. Make a chart on the board or overhead and combine the answers from all groups.

EXPRESSION	SYMBOL
added to plus more than increased by sum	+
subtracted from less than minus decreased by diminished by difference less	−
multiplied by times of product	×
divided by quotient	÷ OR /
the quantity	( )
twice	2 TIMES
half	DIVIDED BY 2
is is the same as equals the result is	=

NOTE TO THE TEACHER: It is better for students to make the chart than have it presented to them by the teacher.

To develop a technique for solving word problems, students should ask themselves the following questions:

1. What am I asked to find?

   Example: **a number**
   first and second number
   larger and smaller number
   three consecutive even integers
   the tens digit and units digit of a number
   the length and width of a rectangle
   the length of the sides of a triangle

   Once the answer to this question is determined, the student should <u>write the unknown down.</u>

   Example: **One number is four more than a second number. Their sum is fourteen. What are the two numbers? Write down what you're asked to find. Let first number = second number =**

2. If I must find more than one quantity, which must I know to find the other?

   In the above example, **"one number is four more than a second number," so** if I know the second, I can find the first. Thus, the second number is "x."
   Write this down.

   Let first number =
   second number = x

3. What is the relationship between the two quantities?

   In the **above example, "one number is four more than a second number," so** the first number is "x + 4." Write this down.

   Let first number = x + 4
   second number = x

4. What is the equation that is the translation of the sentence in the problem?

   In Question 3, **"their sum is fourteen."** The two numbers are $x + 4$ and x. Their sum would be $x + 4 + x$. Is means =. We now have $x + 4 + x = 14$. Write this down.

NOTE TO THE TEACHER: Emphasize that the equation is separate and should be written beneath the "let" statement. The "let" statement, once set up, is never changed.

Let first number = x + 4

second number = x

$$x + 4 + x = 14$$

5. **Solve the equation:**

$$x + 4 + x = 14$$
$$2x + 4 = 14$$
$$2x = 10$$
$$x = 5$$

6. Fill in the blanks with the answers.

Let first number = x + 4 = 9

Second number = x = 5

7. Check your answers with the requirements of the problem. Do your answers make any sense?

F. STUDENT WORKSHEET

WORDS INTO SYMBOLS **Name______________________**

A. Translate the following English phrases into their mathematical equivalents.

1. x plus 12

2. 8 minus y

3. a added to c

4. 4 more than n

5. q less than p

6. twice m

7. y increased by 3

8. 6 subtracted from b

9. x multiplied by y

10. 5 times ab

11. m divided by n

12. the sum of y and z

13. x less 2

14. a decreased by 2

F. STUDENT WORKSHEET (Page 2)

WORDS INTO SYMBOLS **Name______________________**

15. **the product of 2 and m**

16. **6 more than half of w**

17. **negative three-fifths the quantity 7 minus y**

18. **the quotient of m and n**

19. **5 diminished by x**

20. **two-thirds of n**

21. **twice the sum of a and b**

22. **the difference between p and q**

23. **6 times the quantity c minus d**

24. **4 less than the product of a and b**

25. **sum of m & n, divided by 3**

F. STUDENT WORKSHEET (Page 3)

Name____________________________

WORDS INTO SYMBOLS

B. Solve each:

1. **Twelve more than four times a number is four. What is the number?**

2. **Find a number such that twice the number increased by seven is the same as four times the number decreased by three.**

3. **The larger of two numbers is five more than the smaller. The smaller multiplied by -3 is equal to one less than five times the larger. Find the numbers.**

4. **The first of two numbers is 4 more than half of the second. Twice the first added to five is seven less than three times the second. What are the numbers?**

5. **Find three consecutive integers such that three times the largest decreased by the smallest is fifty.**

F. <u>STUDENT WORKSHEET</u> (Page 4)

WORDS INTO SYMBOLS **Name__________________________**

6. **Twice the second of four consecutive odd integers is three less than the fourth. Find the integers.**

7. **The units digit of a two-digit number is three more than the tens digit. If the tens digit is multiplied by three, the result is one less than twice the units digit. What is the number?**

8. **The length of a rectangle is 6 less than three times the width. The perimeter of the rectangle is 52 cm. Find the length and the width.**

9. **The width of a rectangle is 3 less than twice the length. The rectangle has an area of 35 feet squared. What are the length and width?**

10. **The perimeter of a triangle is 127 cm. The first side is three more than twice the second side. The third side is 20 cm more than the second side. Find the measure of all three sides of the triangle.**

<u>Program Ten Words Into Symbols</u>

Student Worksheet F

A. 1) $x + 12$ 2) $8 - y$ 3) $a + c$ 4) $4 + n$ 5) $p - q$ 6) $2m$

7) $y + 3$ 8) $b - 6$ 9) xy 10) $5ab$ 11) $\frac{m}{n}$ 12) $y + z$

13) $x - 2$ 14) $a - 2$ 15) $2m$ 16) $6 + \frac{1}{2}w$ 17) $-\frac{3}{5}(7 - y)$

18) $\frac{m}{n}$ 19) $5 - x$ 20) $\frac{2}{3}n$ 21) $2(a + b)$ 22) $p - q$

23) $6(c - d)$ 24) $ab - 4$ 25) $\frac{m + n}{3}$

B. 1) $12 + 4x = 4$

$x = -2$

2) $2n + 7 = 4n - 3$

$n = 5$

3) $-3x = 5(x + 5) - 1$

$x = -3$ The two numbers are –3 and 2.

4) $2(4 + \frac{1}{2}x) + 5 = 3x - 7$

$x = 10$ The two numbers are 10 and 9.

5) x, $x + 1$, $x + 2$ are the three consecutive integers

$3(x + 2) - x = 50$

$x = 22$ The three consecutive integers are 22, 23, and 24.

6) $x, x + 2, x + 4, x + 6$ are the four consecutive odd integers

$2(x + 2) = x + 6 - 3$

$x = -1$ The four consecutive odd integers are –1, 1, 3, and 5.

7) x is tens digit and $x + 3$ is units digit

$3x = 2(x+3) - 1$

$x = 5$ The number is 58.

8) length $= 3w - 6$ width $= w$

$2(3w - 6) + 2w = 52$

$w = 8$ so length $= 3w-6 = 18$

The width is 8 cm and length is 18 cm.

9) width $= 2L - 3$ length $= L$

$L(2L-3) = 35$

$2L^2 - 3L = 35$

$2L^2 - 3L - 35 = 0$ Solve by factoring.

$(2L + 7)(L - 5)= 0$

$L = \cancel{-\frac{7}{2}}$ or $L = 5$ Since length is a measure of distance, the L can not be negative.

The length is 5 feet and the width is 7 feet.

10) 1^{st} side $= 3 + 2x$, 2^{nd} side $= x$, 3^{rd} side $= 20 + x$

$127 = 3 + 2x + x + 20 + x$

$x = 26$

The 1^{st} side = 55 cm, the 2^{nd} side = 26 cm, and the 3^{rd} side = 46 cm.

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